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ELEMENTARY PROBLEMS IN HAMILTONIAN OPTICS

APRIL 1967

P. J. Nawrocki

Prepared for

SENSORS & ENVIRONMENTAL FACTORS DIVISION

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 1820

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ABSTRACT

Synge's formulation for Hamilton's theory of geometric optics is used to specify the rays for a variety of problems. Based upon the analogy between ray and particle trajectories, an attempt is made to treat the problem of beyond-the-horizon atmospheric propagation. It is shown that the observed propagation law (e^{-ks}) can be motivated from the concept that the received signal is the sum of the Hamilton rays linking the transmitter and receiver. This implies three differences with respect to the Booker-Gordon theory for the everpresent scatter signal: a change in the correlation function from exponential to Gaussian; adoption of millimeters as the size of the characteristic fluctuations; and an extension of the critical volume to include manyfold scatter in the tropospheric path from the optical horizon to the receiver.

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SECTION I

INTRODUCTION

The ray treatment also called geometric optics, is considered to be appropriate when the wavelength is small compared to the characteristic length in slowly varying media comprising the optical instrument, yet large compared to molecular dimensions. Within the realm of geometric optics, there are several possible starting points. For example, a generalized form of the Sommerfeld-Runge relation (equivalent to Snell's formulation) or forms derivable from the Hamilton-Fermat principle (the Hamilton-Jacobi differential-equation formulation or Hamilton's canonical equation) can be employed.

Until recently, the first system suffered because it was very difficult to formulate Snell's law for the most general case of inhomogeneous and anisotropic media. However, this situation has apparently been alleviated since Poeverlein [Ref. 1]* has recently shown that the Sommerfeld-Runge "Law" ($\vec{\nabla} \times \vec{K} - 0$) together with $F(\vec{r}, \vec{k}) = 0$ provide for the rays under a wide range of conditions, including anisotropy and inhomogeneity. Absorption and lateral-intensity variations can be included by introducing complex propagation vectors. Further generalization to four dimensions enables treatment of modulated waves and time-varying media. Once formulated, calculation of the rays should be straight forward.

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* Numbers in bracket designate References listed at end of report.

However, conservation of the tangential components across discontinuities appears to be afflicted with the same limitation to stratified media that plagued the simpler theory.

On the other hand, the Hamiltonian system permits almost routine formulation of the most general problem, and there is no limitation to stratification or to constant B-field. For these reasons, and since Hamilton's original work was performed approximately a century ago, it is difficult to account completely for the fact that Hamilton's optics is not universally taught. Perhaps one reason is that there is no motivation in optics comparable to the role of Hamilton's discipline as a prerequisite for quantum mechanics. However, the power and elegance of Hamilton's optics alone are sufficient justification for more widespread use.

Since the system usually leads to a set of nonlinear second-order differential equations, a computer is the only convenient means for computation of the rays; this, however, is true of any discipline, provided the medium is sufficiently arbitrary. To date, the large digital computer can best handle the anisotropic ray computation. Wong [Ref. 2] has employed the analog computer for the troposphere and is currently synthesizing a program for using this device for the more difficult (inhomogeneous anisotropic) ionosphere. Unfortunately, it appears that without extensive modification the analog computer does not carry enough significant figures to give reproducible results.

For purposes of this report, it is believed that that Hamiltonian system will provide the best means of attacking new problems since the analogous disciplines in mechanics, statistical mechanics, and quantum mechanics can be used as a guide.

SECTION II

RAY TREATMENT

HAMILTONIAN OPTICS FOR CARTESIAN COORDINATE SYSTEMS

For Cartesian coordinate systems the Hamiltonian system as given by Synge [Ref. 3] is particularly simple and direct. Consider a fixed (no time-dependence) optical instrument or an aggregate of media whose phase refractive index is a function of position for the inhomogeneous case and a function of direction for the anisotropic case. Thus $v = v\left(x_k, \alpha_k\right)$, where α_k is the direction cosine. In addition, $\alpha_k \alpha_k = 1$, where the repeated index implies the summation convention.

The Euler-Lagrangian condition that the ray path with fixed end-points must be an extremum, as obtained from the variational principle $\delta \int (1/v) ds = 0$ is expressed by

$$\frac{d}{ds} \frac{\partial}{\partial \alpha_k} \left(\frac{1}{v}\right) - \frac{\partial}{\partial x_k} \left(\frac{1}{v}\right) = 0 . \tag{1}$$

Because of the auxiliary condition $\alpha_k \alpha_k = 1$, it is possible to decide further that the Langrangian $(1/v)$ shall be homogeneous of degree-plus-one in the α_k, Then obviously, $\frac{\partial}{\partial \alpha_k} (1/v)$ is of degree-zero in the α_k. This expression defines the canonical variable of slowness, σ_k .

Figure 1 gives the relation between the ray surface or indicatrix $r = v\left(x_k, \alpha_k\right)$ and the basic wave surface or figuratrix $\Omega\left(x_k, \sigma_k\right) = 0$. These are the reciprocal surfaces of Hamilton and are related by $PS \cdot PR = PS \cdot PQ \cos\theta = 1$.

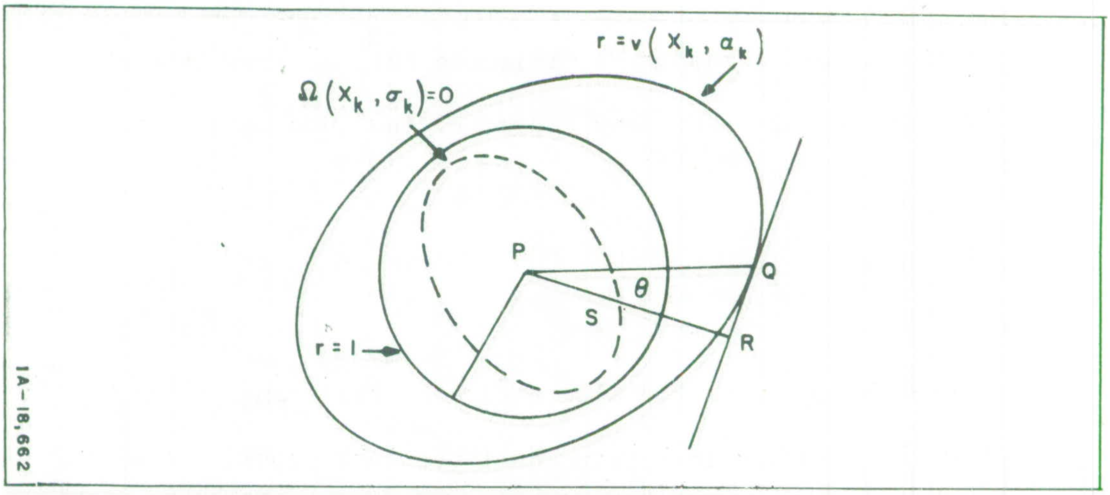

Figure 1. The Reciprocal Wave Surfaces of Hamilton

In principle, the ray paths can be derived from either of the two surfaces; however, one is usually given the phase-refractive index of the medium. Since the figuratrix or ray Hamiltonian is given in terms of the phase refractive index, it is more convenient to employ the ray Hamiltonian to derive the rays. Indeed, vanishing of the ray Hamiltonian and the system of canonically conjugate equations,

$$\frac{\partial\Omega}{\partial\sigma_k} = \frac{dx_k}{du}\ ; \quad \frac{\partial\Omega}{\partial x_k} = -\frac{d\sigma_k}{du}\ , \tag{2}$$

provides a sufficiency condition for determining the rays. The parameter u is related to the time t through the degree of homogeneity of $(\Omega + 1)$ in σ_k, or more explicitly,

$$t = \int \sigma_k \frac{\partial \Omega}{\partial \sigma_k} du .$$

The canonical equations (Equation (2)) are supplemented, in the case of discontinuous media, by the jump conditions

$$\Delta\sigma_k \cdot \delta x_k = 0 . \tag{3}$$

In addition, there is the alternative of replacing the discontinuity with a thin continuous layer that provides the appropriate ray trajectory consistent with Equations (2) and (3).

GENERATING FUNCTIONS OF THE RAY HAMILTONIAN

The generating function (Synge) appropriate to both inhomogeneous and anisotropic media, but restricted to Cartesian coordinate systems is given by

$$\Omega(x_k, \sigma_k) = \frac{\sqrt{\sigma_k \sigma_k}}{f\left(x_k, \sigma_1/\sigma_2, \sigma_3/\sigma_2\right)} - 1 = 0 , \tag{4}$$

where $c = 1$ throughout. In this case $\Omega + 1$ is homogeneous of degree-plus-one in the σ_k, so that, further, $u = t$. To be able to employ orthogonal systems other than the Cartesian system, it is acceptable to set up the ray Hamiltonian in the above form and then to transform coordinates in the final system of differential equations.

In the special case of isotropic media, the ray Hamiltonian for all orthogonal coordinate systems is given by (Synge)*

$$g^{ik} \sigma_i \sigma_k - n^2(x_k) = 0 , \tag{5}$$

where n is the phase-refractive index and g^{ik} is the contravariant metric fundamental tensor defined by the familiar relations

$$g^{ik} = \frac{\text{minor of } g_{ik}}{|g_{ik}|} , \quad ds^2 = g_{ik} \, d\xi^i \, d\xi^k .$$

*Private correspondence with the author.

The Appleton-Hartree-Goubauian formulation of the refractive index for the ionosphere is given by

$$n = \mathrm{Re}\left\{ 1 - \frac{x}{1 - \frac{\nu}{\omega} + \ell x - \frac{y_T^2}{2\left(1 - \frac{x}{n_i^2} - i\frac{\nu}{\omega}\right)} \pm \left[\frac{y_T^4}{4\left(1 - \frac{x}{n_i^2} - \frac{i\nu}{\omega}\right)} + y_L^2\right]^{1/2}} - \frac{x_i^2}{1 - \frac{i\nu_i}{\omega}} \right\}^{1/2} . \qquad (6)$$

If the ion contribution, the collision frequency, and the Lorentzian polarization term can be ignored, the refractive index reduces to

$$n^2 = 1 - \frac{1}{\alpha - \frac{y_T^2}{2(1+\alpha)} \pm \left[\frac{y_T^2}{4(1+\alpha)} \quad y_L^2\right]^{1/2}} , \qquad (7)$$

and the ray Hamiltonian describes an e psoid of revolution whose axis of symmetry is along the terres al magnetic-field lines. If the further restriction is made t radio frequencies are in excess of median MUF_s (maximum usable frequency for path s), then $\omega^2 \gg \omega_H^2$ and the expression for the index reduces to

$$n^2 = 1 - \frac{4\pi N_e^2}{m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} . \qquad (8)$$

In this case, the ellipsoidal surface defining the ray Hamiltonian degenerates to a sphere appropriate to isotropic media.

NOTE

It is now customary to ignore the Lorentzian polarization term ($4/3\ \pi P$) in the contribution of free electrons to the refractive index of the ionosphere. It is included in the general expression, Equation (7), since it is evident that the term is pertinent to the ion contribution. The Lorentzian term takes into account the self-field of the specie as polarized by the external field. The electron, a monopole with a dimension of the order of 10^{-13} cm, moves with ease to minimize the distortion. In the case of the ion (a complex ion such as O^+, and not H^+) the charge separation (of the order of 10^{-8} cm) and the relative immobility of the ion permit the specie to be polarized to such a degree that the self-field is an important contribution.

THE TENSOR FORM OF THE VARIATIONAL PRINCIPLE

The presentation of Hamilton's system as formulated by Synge began with an expression of the variational principle which, while obviously applicable to inhomogeneous and anisotropic media, was limited to Cartesian coordinate systems. For the most general media it was possible to take advantage of preferred coordinate systems or ignorable coordinates only by finally transforming the

system of differential equations. Such a scheme does not take full advantage of the ignorable coordinate. Haselgrove [Ref. 4] has demonstrated that a start can be made with a tensor expression of the variational principle which is appropriate to all orthogonal coordinate systems and, of course, to both anisotropic and inhomogeneous media. By replacing total and partial derivatives by terms involving the covariant derivative, Equation (1) becomes

$$\frac{\partial}{\partial x_i}\left(\frac{1}{G}\right) - u^{\ell}\frac{\partial}{\partial u^m}\left(\frac{1}{G}\right)\{\ell i, m\}$$

$$- g_{ij}\left[\frac{d}{dt}\left(\frac{u^i}{G}\right) + \frac{u^{\ell}}{G}\frac{dx_k}{dt}\{\ell k, j\}\right] = 0\,, \qquad (9)$$

where the reciprocal surfaces of Hamilton are $F = 1$ and $G = 1$; $\{\ell i, m\}$ denotes the familiar Christoffel symbol of the second kind; and x_i and u^i donote the position and velocity, respectively. The double notation $\left(\vec{r} = \vec{v},\ \Omega = 0;\ F = 1,\ G = 1\right)$ has been retained because the surface can be expressed either in terms of the slowness vector or its inverse, the velocity vector. Hamilton's

canonical forms can then be written as

$$\frac{dx_j}{dt} = g^{ij} \frac{\partial G}{\partial u^i}$$

$$\frac{du^j}{dt} = g^{ij} \frac{\partial G}{\partial x_i} - g^{ij} g^{km} u^\ell \frac{\partial G}{\partial u^m} \left(\frac{\partial g_{\ell i}}{\partial x_k} - \frac{\partial g_{k\ell}}{\partial x_i} \right) \quad . \tag{10}$$

Equations (10), together with the basic wave surface $G = 1$ define the rays for any coordinate system. For the spherically symmetric case the six Equations (1) reduce (Haselgrove) to

$$\frac{dr}{dt} = \frac{1}{n^2} \left(v^r - n \frac{\partial n}{\partial v^r} \right)$$

$$\frac{d\theta}{dt} = \frac{1}{rn^2} \left(v^\theta - n \frac{\partial n}{\partial v^\theta} \right)$$

$$\frac{d\Phi}{dt} = \frac{1}{rn^2 \sin\theta} \left(v^\Phi - n \frac{\partial n}{\partial v^\Phi} \right)$$

$$\frac{dv^r}{dt} = \left(\frac{1}{n} \frac{\partial n}{\partial r} + v^\theta \frac{d\theta}{dt} + \sin\theta \, v^\Phi \frac{d\Phi}{dt} \right)$$

$$\frac{dv^\theta}{dt} = \frac{1}{r} \left(\frac{1}{n} \frac{\partial n}{\partial \theta} - v^\theta \frac{dr}{dt} + r \cos\theta \, v^\Phi \frac{d\Phi}{dt} \right)$$

$$\frac{dv^\Phi}{dt} = \frac{1}{r \sin\theta} \left(\frac{1}{n} \frac{\partial n}{\partial \Phi} - \sin\theta \, v^\Phi \frac{dr}{dt} - r \cos\theta \, v^\Phi \frac{d\theta}{dt} \right) \quad .$$

(11)

In spite of their tensor form, Equations (9) and (10) do not define a Lorentz covariant theory. This is clear because the frequency enters only in determination of the refractive index n and not as a variable having the status of x_j , u^j . A four-dimensional discipline, allegedly appropriate to a wider variety of problems, has been formulated by Synge (see pages 25 to 28). The Haselgrove discipline could be employed for some problems by taking the refractive index at the appropriate value of the Doppler frequency as given by relatively.

THE SPHERICALLY-SYMMETRIC INHOMOGENEOUS ISOTROPIC CASE

In Hamiltonian optics, as in Hamiltonian mechanics, the coordinate system is chosen to take advantage of the existence of ignorable coordinates, since the latter lead immediately to constants of the ray or particle trajectory. Invariably, the canonical conjugate of an ignorable coordinate is a constant of the motion. In the isotropic case the ray Hamiltonian is given by Equation (5), and the coordinate system is chosen so that g_{ik} and n involve the smallest number of coordinates. The selected differential line element is then $ds^2 = g_{ik}\ d\xi^i\ d\xi^k = dr^2 + r^2 d\theta^2 + r^2 \sin\theta\ d\Phi^2$, and the ray Hamiltonian takes the form (again $c = 1$)

$$\Omega = g^{ik}\,\sigma_i \sigma_k - n^2(x_k) = 0$$

$$\Omega = \sigma_r{}^2 + \frac{1}{r^2}\,\sigma_\theta{}^2 + \frac{1}{r^2 \sin^2\theta}\,\sigma_\Phi{}^2 - n^2(r) = 0\,. \tag{12}$$

The origin is, of course, coincident with the center of symmetry. It is worth repeating that vanishing of the ray Hamiltonian and the canonical equations is sufficient to uniquely define the ray trajectories. The canonical Equations (2) give

$$\frac{dr}{du} = 2\sigma_r$$

$$\frac{d\theta}{du} = \frac{2\sigma_\theta}{r^2}$$

$$\frac{d\Phi}{du} = \frac{2\sigma_\Phi}{r^2 \sin^2\theta} = \frac{2C_1}{r^2 \sin^2\theta}$$

$$\frac{d\sigma_r}{du} = \frac{2\sigma_\theta{}^2}{r^3} + \frac{2\sigma_\Phi{}^2}{r^3 \sin^2\theta} + 2n\,\frac{dn}{dr}$$

$$\frac{d\sigma_\theta}{du} = \frac{2\sigma_\Phi{}^2 \cos\theta}{r^2 \sin^3\theta}$$

$$\frac{d\sigma_\Phi}{du} = 0\,;\quad \sigma_\Phi = C_2\,. \tag{13}$$

As anticipated, the coordinate σ_{Φ} canonically conjugated to the ignorable coordinate Φ is a constant of the ray trajectory. In addition, inspection of the homogeneity of

$$\Omega\,(r, \theta, \sigma_r, \sigma_\theta, \sigma_\Phi) = 0 \text{ in } \sigma_k$$

indicates that the phase parameter $u = t/2$. The number of equations can be reduced by eliminating the σ_k, although this form is not necessarily more advantageous for computer use.

Setting $P = \frac{dr}{du}$, $Q = \frac{d\theta}{du}$, $R = \frac{d\Phi}{du}$ produces

$$\frac{dP}{du} = rQ^2 + \frac{4C_1^2}{r^2 \sin^2\theta} - 4n\frac{dn}{dr}$$

$$\frac{d}{du}(r^2 Q) = \frac{4C_1^2 \cos\theta}{r^2 \sin^3\theta}$$

$$R = \frac{2C_1}{r^2 \sin^2\theta} \qquad (14)$$

For application to a spherical electron cloud with radial symmetry, solution of Equation (14) yields a three-dimensional picture of the rays. The simultaneous solution of properly selected rays, together with the equation of the terrestrial surface, can also provide an estimate of the ray pattern or relative intensity of radiation on the ground. The effect of absorption along the ray trajectory in the ionized cloud can also be included.

If infinite skew-rays are ignored the problem of defining ray paths in a spherically-symmetric medium (ion cloud) can also be solved in two dimensions by taking a planar cut through the cloud. The plane is defined by two coincident lines: the initiating ray, and a line between the point of ray initiation and the center of symmetry. Polar coordinates are again selected to take advantage of the existence of an ignorable coordinate, but it is to be emphasized that only the θ coordinate has meaning in a particular plane. The ray Hamiltonian becomes

$$\sigma_r^2 + \frac{1}{r^2}\,\sigma_\theta^2 - n^2(r) = 0\,, \tag{15}$$

and the Hamiltonian canonical equations give the four equations

$$\frac{d\sigma_r}{du} = \frac{2}{r^3}\,\sigma_\theta^2 + 2n\,\frac{dn}{dr}$$

$$\frac{d\sigma_\theta}{du} = 0\,; \quad \sigma_\theta = C_3$$

$$\frac{dr}{du} = 2\sigma_r$$

$$\frac{d\theta}{du} = \frac{2C_3}{r^2} \,. \tag{16}$$

The homogeneity of the ray Hamiltonian in σ_k has not changed, so $t = 2u$. Substituting for the σ_k and $\dot{\sigma}_k$ immediately produces

$$\frac{d}{du}\left(\frac{dr}{du}\right) + 2C_3{}^2 \frac{d}{dr}\left(\frac{1}{r^2}\right) - 2\frac{d}{dr}(n^2) = 0 . \tag{17}$$

But, since θ is ignorable, $d/du = (dr/du)/\ d/dr$. Integrating

$$\left(\frac{dr}{du}\right)^2 + 4C^2 r^{-2} - 4n^2 = 0 , \tag{18}$$

and finally for the ratio of $d\theta/du$ to dr/du

$$\frac{d\theta}{dr} = r^{-1}(Cn^2 r^2 - 1)^{-1/2} . \tag{19}$$

It is well known that this expression can also be obtained from the cylindrical form of Snell's Law ($nr \sin \theta = k$), but not with the facility and generality characteristic of the Hamilton system.

RAY PATHS IN ISOTROPIC MEDIA WITH SPHEROIDAL SYMMETRY

This particular example is concerned with inhomogeneous media having equirefractive index surfaces that are prolate or oblate spheroids (ellipsoids of revolution). At first glance it might appear that ellipsoidal coordinates are the most advantageous; however, for an ignorable coordinate to exist in the ray Hamiltonian,

neither the g_{ik} nor the n can contain the coordinate. To satisfy this criterion the spherical coordinate system is again selected, and the ray Hamiltonian is given by

$$\Omega = g^{ik} \sigma_i \sigma_k - n^2 (x_k) = 0$$

$$= \sigma_r^2 + \frac{1}{r^2} \sigma_\theta^2 + \frac{1}{r^2 \sin^2 \theta} \sigma_\Phi^2 - n^2 (r, \theta) = 0 . \tag{20}$$

The functional dependence of the refractive index upon r, θ (Φ being ignorable by rotational symmetry) is (e.g., artificial cloud)

$n^2 = 1 - A \exp \left\{ - \left[r \frac{(1 - \epsilon \cos \theta)}{\beta} \right]^2 \right\}$ for the oblate spheroid, and

$n^2 = 1 - A \exp \left\{ - \left[r \frac{(1 - \epsilon \sin \theta)}{\beta} \right]^2 \right\}$ for the prolate spheroid. Again, vanishing of the ray Hamiltonian and Hamilton's canonical equations is sufficient to determine the ray paths as expressed in Equation (21):

$$\frac{d\sigma_r}{du} = \frac{2\sigma_\theta^2}{r^3} + 2 \frac{\sigma_\Phi^2}{r^3 \sin^2 \theta} + 2n \frac{\partial n}{\partial r}$$

$$\frac{d\sigma_\theta}{du} = \frac{2 \cos \theta}{r^2 \sin^3 \theta} \sigma_\Phi^2 + 2n \frac{\partial n}{\partial \theta}$$

$$\frac{d\sigma_\Phi}{du} = 0 ; \quad \sigma_\Phi = C$$

$$\frac{dr}{du} = 2\sigma_r$$

$$\frac{d\theta}{du} = \frac{2\sigma_\theta}{r^2}$$

$$\frac{d\Phi}{du} = \frac{2\sigma_\theta}{r^2 \sin^2\theta} \quad . \tag{21}$$

Inserting the values of σ_k and $\dot{\sigma}_k$, and, further, letting $P = dr/du$, $Q = d\theta/du$, and $R = d\Phi/du$, the system of equations reduces to

$$\frac{dP}{du} = rQ^2 + \frac{4C^2}{r^3 \sin^2\theta} + 4n\frac{\partial n}{\partial r}$$

$$\frac{d}{du}(r^2 Q) = \frac{4C^2 \cos\theta}{r^2 \sin^3\theta} + 4n\frac{\partial n}{\partial \theta}$$

$$\frac{d\Phi}{du} = \frac{2C}{r^2 \sin^2\theta} = R \ . \tag{22}$$

INHOMOGENEOUS AND ANISOTROPIC RAYS IN MEDIA WITH NO SYMMETRY

The system of differential equations that can yield ray paths for any continuous medium to which geometric optics is appropriate and as given by Haselgrove, was specified in Equations (9) and (10) for any coordinate system and in Equation (11) for polar coordinates.

Generally speaking, an anisotropy in an ionized gas is a result of the impressed magnetic field. Moreover, the field lines are parallel over the extent of the plasma of interest. It is quite natural, then, for the specification of the ray Hamiltonian in the form of an ellipsoid of revolution to be simplest in Cartesian coordinates where one of the coordinates (say z) is parallel to the field vector. The ray Hamiltonian is then given simply as

$$\Omega(x_k, \sigma_k) = \frac{\sigma_x^{\,2}}{a^2(x_k)} + \frac{\sigma_y^{\,2}}{b^2(x_k)} + \frac{\sigma_z^{\,2}}{c^2(x_k)} - 1 = 0 . \qquad (23)$$

Again, vanishing of the ray Hamiltonian and Hamilton's canonical equations provides a sufficiency condition for determining the ray trajectories.

Substituting the values of σ_k and $\dot{\sigma}_k$, the cannonical equations lead to the system of nonlinear second-order differential equations

$$\frac{dP}{du} = f_1(x_k)\,P^2 + g_1(x_k)\,Q^2 + h_1(x_k)\,R^2$$

$$\frac{dQ}{du} = f_2(x_k)\,P^2 + g_2(x_k)\,Q^2 + h_2(x_k)\,R^2$$

$$\frac{dR}{du} = f_3(x_k)\,P^2 + g_3(x_k)\,Q^2 + h_3(x_k)\,R^2 . \qquad (24)$$

The system can also include time-dependent media. In this case, there is an added term in Equation (24) of the form P d/du (f_o), etc. An unsuccessful attempt was made to linearize the system of equations so that the modified Peano-Baker matrizant methods could be applied, although the system in its existing form is quite amenable to computer techniques.

ABSORPTION, PHASE, AND TIME

In the ray treatment to this point, absorption of energy has not been considered. To include absorption, the time increment of group propagation must be known, i.e., time-reckoning along the ray path. This introduces an added element of complexity because the presence of the phase refractive index in the previous equations implies that the time-reckoning involved is the phase (periodicity of the wave). The true time is then given in terms of the phase propagation time, t, as

$$dt' = \frac{n_{group}}{n_{phase}}\, dt = \frac{n + \omega \dfrac{\partial n}{\partial \omega}}{n}\, dt \,. \tag{25}$$

Haselgrove conveniently includes the phenomenon of absorption by introducing the above equation, together with

$$\frac{dD}{dt} = -\frac{\mathit{n}}{n}\, D \,, \tag{26}$$

where $\kappa = \kappa\ (x_k, u^k)$ is the absorption coefficient, $t'(c)$ is the equivalent path length in vacuo, and D is the absorption $D = \exp \{ - \int \kappa \cos \theta ds \}$. The phase t is consistently used throughout; and the simultaneous solution of Equations (9), (10), (25), (26) gives all the required parameters of propagation in the most general case of time-stationary media.

Parenthetically, the confusion between phase t and time t' extends through much of physics. For example, the t used in most of quantum theory is phase not time. As motivation for this assertion, first quantization confers a wave property to the particle (defined here as any simply connected intracule or any extracule, always of nonvanishing rest mass). This is illustrated by experiments on a queue of electrons impacting on a microscopic double slit. Intuitively, the electron passes through one slit or the other. However, the accompanying phase wave, propagating at a velocity in excess of the characteristic velocity c permits the electron to be aware of the existence of both apertures and, via this phase wave, to interfere with itself.

It might be argued that quantum mechanical space is everywhere homogeneous and isotropic [see Equations (28), (29)]. The temporal reckonings in phase t and time t', therefore, differ only by a constant factor that cannot affect the essential phenomenology. However, Eddington [Ref. 5] has asserted that the difference is

indeed significant. Quantum mechanics is a particle theory, i.e., one in which the perturbation between the object system and the environment is linked to the object system, the environment remaining essentially invariant (the Hartree-Fock self-consisten field). Consequently, it is conceptually possible that, in certain cases, the perturbation of this so-called unpolarizable environment might be sufficiently strong to make the postulate of isotropic homogeneous space rather unrealistic and, therefore, a relatively inconvenient starting point.

In other words, a reduced electron harmonically oscillating in a radial potential well may present an environment quite different from that of a reduced nucleon also harmonically oscillating in the nucleus. Consequently, interpretation of the quantum-mechanical t may actually depend upon the physical problem, and may become a vital consideration in nucleon phenomenology.

If further motivation for the existing state of affairs is required, historically, the most vital problem solved by the quantum mechanist is $H\Psi = E\Psi$; this defines stationary states in which the phase (and/or time) is deliberately left unspecified. Time may be pertinent in the case of an electron (when one could substitute the Fourier transformation of the moving charge in real time), but this pertinence does not extend to a neutron moving in a potential well. Consequently, it would appear that the theorists have not fully examined the temporal ramifications of postulating that particles

obey such wave equations as the Klein-Gordon equation

$$(\square^2 + m^2)\, \Psi(\vec{x}, t) = 0 , \tag{27}$$

where the further postulated <u>periodic</u> nature of the temporal parameter implies phase not time.

THE OPTICAL-MECHANICAL ANALOGY

In the previous problems, the analogy between optical and mechanical solutions is most striking. Thus, the Hamiltonian for the mechanical particle in spherical coordinates, $\frac{1}{2m}\left(p_r{}^2 + p_\theta{}^2 + p_\Phi{}^2\right) + V = H$, on substitution of the operator, $p \to -i \frac{\partial}{\partial q}\ (\hbar = 1)$, becomes, in $H\Psi = E\Psi$

$$\left[\left(\partial_r{}^2 + \frac{1}{r^2}\, \partial_\theta{}^2 + \frac{1}{r^2 \sin^2\theta}\, \partial_\theta{}^2\right) - 2mE\right] \Psi = 0 , \tag{28}$$

closely analogous to the isotropic ray Hamiltonian

$$\sigma_r{}^2 + \frac{1}{r^2}\, \sigma_\theta{}^2 + \frac{1}{r^2 \sin^2\theta}\, \sigma_\Phi{}^2 - n^2(r, \theta, \Phi) = 0 . \tag{29}$$

In addition to delineating the role of spatial isotropy in quantum theory the example suggests an expanded analogy as, for example, in Table I.

TABLE I

Optical Mechanical Analogy

Mechanical Systems (holonomic)	Optical Systems
Canonical conjugates generalized position . . . q_k generalized momentum . . . p_k $p_k = \frac{\partial L}{\partial \dot{q}_k}$	Canonical conjugates position . . . x_k "slowness" . . . σ_k $\sigma_k = \frac{\partial}{\partial \alpha_k}\left(\frac{1}{v}\right)$
Time . . . t	Temporal parameter . . . u phase $t = \int \sigma_k \frac{\partial \Omega}{\partial \sigma_k}\, du$ time $t' = \int \left(n + \omega \frac{\partial n}{\partial \omega}\right)\left(\frac{dt}{n}\right)$
Hamiltonian . . . H $H = H(p_k q_k, E, t)$ for time stationary systems $\frac{dH}{dt} = 0$	Ray Hamiltonian . . . Ω $\Omega = \Omega(x_k, \sigma_k, \omega, t)$ for time stationary media $\frac{d\Omega}{dt} = 0$
Canonical equations $\frac{\partial H}{\partial p_k} = \dot{q}_k ; \quad \frac{\partial H}{\partial q_k} = -\dot{p}_k$ Lagrangian $L(q_k, \dot{q}_k) = T - V$ $\delta \int L dt = 0$	Canonical equations $\frac{\partial \Omega}{\partial \sigma_k} = \dot{x}_k ; \quad \frac{\partial \Omega}{\partial x_k} = -\dot{\sigma}_k$ Indicatrix $v(x_k, \alpha_k) = 1$ $\delta \int \left(\frac{1}{v}\right) ds = 0$
Generating function of the Hamiltonian $H = p_k \dot{q}_k - L$	Generating function of the ray Hamiltonian (Figuratrix) $\Omega = \frac{\sqrt{\sigma_i \sigma_i}}{f(x_k, \sigma_1/\sigma_2, \sigma_3/\sigma_2)} - 1 = 0$
	from Atmospheric Processes, Nawrocki & Papa

There is some ambiguity in selecting the analogous terms. This may come about as a result of over-defining the optical system by the relation $E = \hbar\omega$. Thus, the equations of motion involving the two mechanical canonical sets $\left(p_k, q_k \text{ and } H, t\right)$ can actually yield, in the optical case, only a unique ray trajectory. Pertinent to this ambiguity, Landau and Lipshitz apparently prefer to use as the canonical relations

$$\dot{k}_i = -\frac{\partial \omega}{\partial x_i} \; ; \quad \dot{x}_i = \frac{\partial \omega}{\partial k_i} \,, \tag{30}$$

where

$\vec{x}$ is position

$\vec{k}$ is the wave vector $(k\,\vec{n})$.

Clearly, the wave vector, $\vec{k}$, and the frequency, ω , transform like momentum $\left(\vec{p} = \frac{\hbar\omega\vec{n}}{c} = \hbar\vec{k}\right)$ and energy $(E = \hbar\omega)$, respectively. Using this system Landau states

> "However, it (the least action principle) cannot be written as $\delta\int L\,dt = 0$, since it turns out to be impossible to introduce, for rays, a function analogous to the Lagrangian of a particle (i.e.,
>
> $L = \vec{p} \cdot \frac{\partial H}{\partial \vec{p}} - H \rightarrow \vec{k} \cdot \frac{\partial \omega}{\partial \vec{k}} - \omega \equiv 0$), since $\omega = ck$ in volume."

While this is true in the sense specified by Landau , Synge [Ref. 6] has shown that the optical Lagrangian can be formulated provided that a slightly different procedure is followed. He starts with $H\ (p_k, q_k, t)$, defines $\dot{q}_\rho = \frac{\partial H}{\partial p_\rho}$, solves for p_ρ, and defines L as

$$L\ (q_k,\dot{q}_k,t) = p_\rho\,\dot{q}_\rho - H\,(p_k,q_k,t)$$

Thus, Hamilton's principle for the photon becomes

$$\delta \int (\sigma_k\, dx_k + i^2\, Hdt - i^2\, Hdt) = \delta \int \sigma_k\, dx_k = 0\ , \tag{31}$$

or essentially, $\delta \int \left(\frac{1}{v}\right) ds = 0$, i.e., Fermat's Principle.

The complete tabulation of the analogy as specified by Synge, where $c = h = 1$, is as shown in Table II.

TABLE II

Optical Mechanical Analogy as Specified by Synge

Term	Optical	Mechanical
Coordinates	x_p	q_ρ
Time	x_4 (= it)	t
Momentum	σ_ρ	p_ρ
Energy	$\sigma_4 = iH$	H
Hamiltonian	$\Omega = \sigma_4 - iH(\sigma_\rho, x_k) = 0$	$H = H(p_\rho, q_k, t)$
Equations of motion		
	$\frac{idx_\rho}{dx_4} = i \frac{\partial\Omega/\partial\sigma_\rho}{\partial\Omega/\partial\sigma_4} = \frac{\partial H}{\partial\sigma_\rho}$	$\dot{q}_\rho = \frac{\partial H}{\partial p_\rho}$
	$\frac{id\sigma_\rho}{dx_4} = -i \frac{\partial\Omega/\partial x_\rho}{\partial\Omega/\partial\sigma_4} = -\frac{\partial H}{\partial x_\rho}$	$\dot{p}_\rho = -\frac{\partial H}{\partial q_\rho}$
Hamilton Variational Principle	$\delta \int \sigma_k \, dx_k = 0$	$\delta \int (p_\rho \dot{q}_\rho - H) \, dt = 0$
	$\rightarrow \frac{d}{ds} \frac{\partial f}{\partial \alpha_k} - \frac{\partial f}{\partial x_k} = 0$	
	(Euler-Lagrange)	

By introducing the frequency ω as the fourth member in the slowness-frequency tetrad, Synge has demonstrated that, for photon dynamics, an essentially Lorentz-covariant formalism can be developed to solve such problems as

"1. A set of unaccelerated media, separated by vacuum, the optical properties of each medium (when at rest) being known, the media may be heterogeneous and anisotropic.

2. As above, but without the vacuum separation, the media sliding by one another in contact.

3. Fluids in accelerated motion, assuming them optically isotropic in the local or rest frame."

While it is acknowledged that optical science is indebted to Synge's work and there is great admiration for his four-dimensional construct, application of the latter is limited by the dynamic character of optical media. In other words, all ponderable media are composed of dynamic mass and charge-bearing corpuscles whose collective behavior determines the optics. As a result of these mechanical aspects strains, slippage, and ruptures occur which cannot readily be included in the general covariant manner appropriate to acceleration.

SECTION III

SCATTER THEORY OF MICROWAVE PROPAGATION

STATISTICAL OPTICS

The idea of developing a statistical theory of rays originated with the phenomenon of beyond-the-horizon propagation of microwaves. Indeed, most of this section was written in 1954 as the result of a suggestion by T. F. Rogers to seek a solution (alternative to the Booker-Gordon [Ref. 7] solution) that would provide an everpresent transmitted-signal level beyond that predicted by classical propagation theory. This was achieved, but the theory lacked the sophistication necessary to provide for such important parameters as bandwidth, fading rates, and frequency dependence.

The subject of this paper is Hamilton optics, and a thorough review of the scatter theory herein would not be justified. Such reviews have been made by Staras and Wheelon [Ref. 8], Staras [Ref. 9], Shkarofsky [Ref. 10] and Vysokovskie [Ref. 11]. However, existing scatter theory has marked capabilities and it would be a misrepresentation to dwell only on its failures; consequently, a very brief review of the salient features is in order.

As with most controversial areas, the experimental evidence varies to some extent with the investigator. Again, there is no attempt to weight conflicting numbers, and the data presented must reflect only casual acquaintance with this area.

1. <u>Distance Dependence</u>

The field intensities as a function of distance are given in Figure 2, as determined by T. F.Rogers et al [Ref. 12]. The straight line reveals the author's opinion that the mean propagation loss is indeed exponential (e^{-ks}), but other authors are just as convinced that an r^{-n} law is revealed by the same data.

Ninety percent of the time, the median field strength is approximately 57 db below free space at 100 miles, with a further loss of 12 db/100 miles. For one percent of the time, the attenuation relative to free space is 85 db at 100 miles and again increases at 0.1 db per mile.

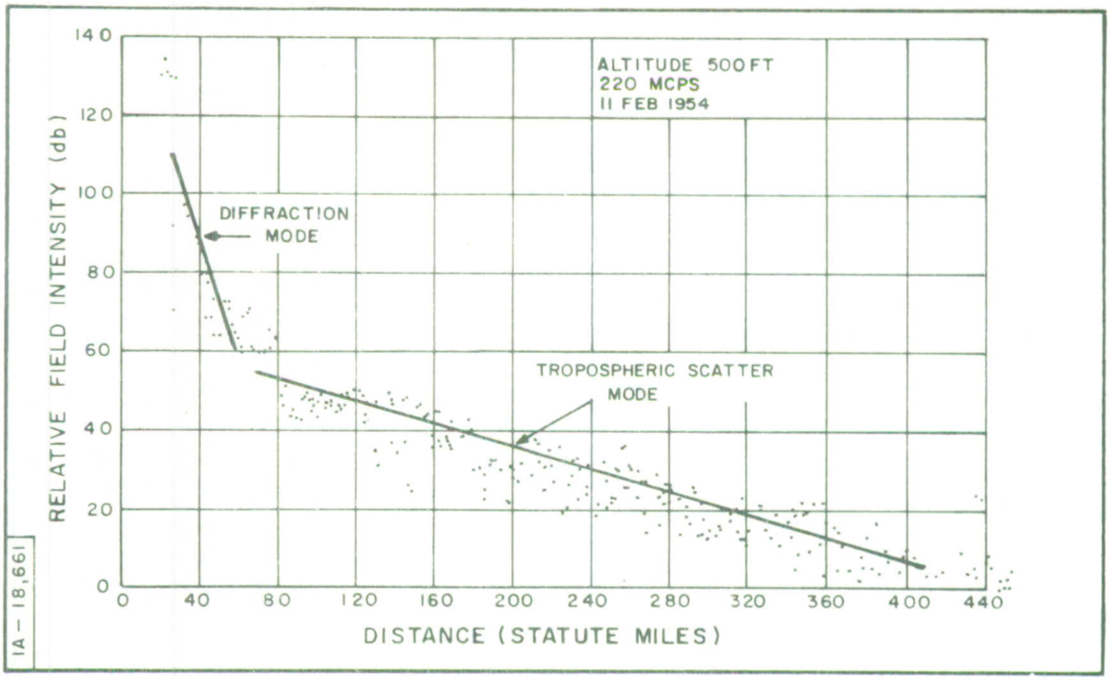

Figure 2. Field Intensities for Tropospheric Scatter as a Function of Distance. (after T. F. Rogers et al)

2. Frequency Dependence

Some estimates are given in Table III but the author adopts Bolgiano's [Ref. 13] estimate that the frequency dependence varies in time (and of course as the meteorological character of the air mass) as indicated by Figures 3 and 4.

Bolgiano also indicates (Figure 5) a relation between wavelength dependence and dynamic stability. To exclude this would again be deliberate misrepresentation, but, at this juncture, the author's theory is not sufficiently sophisticated to make quantitative use of the dynamic character of the air mass.

3. Fading Rate and Bandwidth

As indicated in Table 2, there are two characteristic fading rates: a fast component, 0.1 to 10 c/s, which shall be related to the time required to alter n fluctuations (with scale of the order of millimeters) by collisional processes; and a slow component, 10 minutes to 1 hour, which shall be related to dynamic meteorologic processes (windblown clouds, turbulence, etc.). The bandwidth will not be considered at this initial stage.

Existing Theory of TSM (Tropospheric Scatter Mode)

To date, theories proposed to explain the TSM phenomenon have been based on one of two basic concepts.

1. The received signal is due to scattering from turbulence-produced inhomogeneities in that medium common to the transmitting and receiving beams.

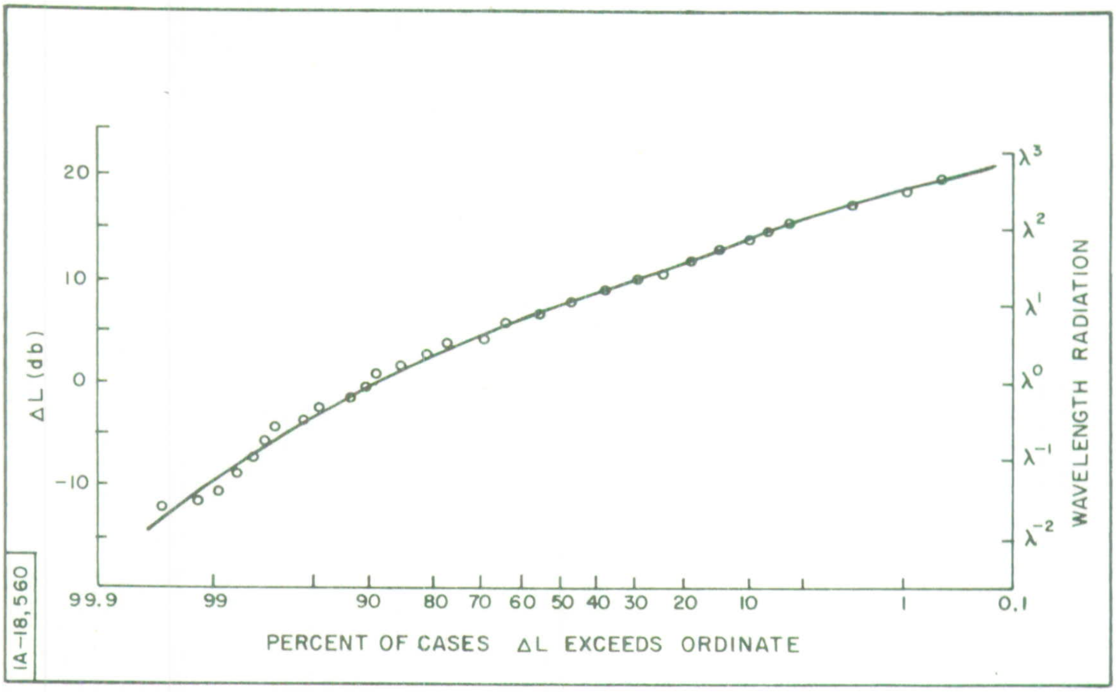

Figure 3. Distribution of Wavelength Dependence

Differences in measured hourly median basic transmission loss, L, scaled frequency experiment, 417.05 Mc/s versus 2290 Mc/s. Round Hill-Crawford's Hill February 11, 1957 - July 11, 1957, 241 cases (after Chisholm, Roche, and Jones, 1957).

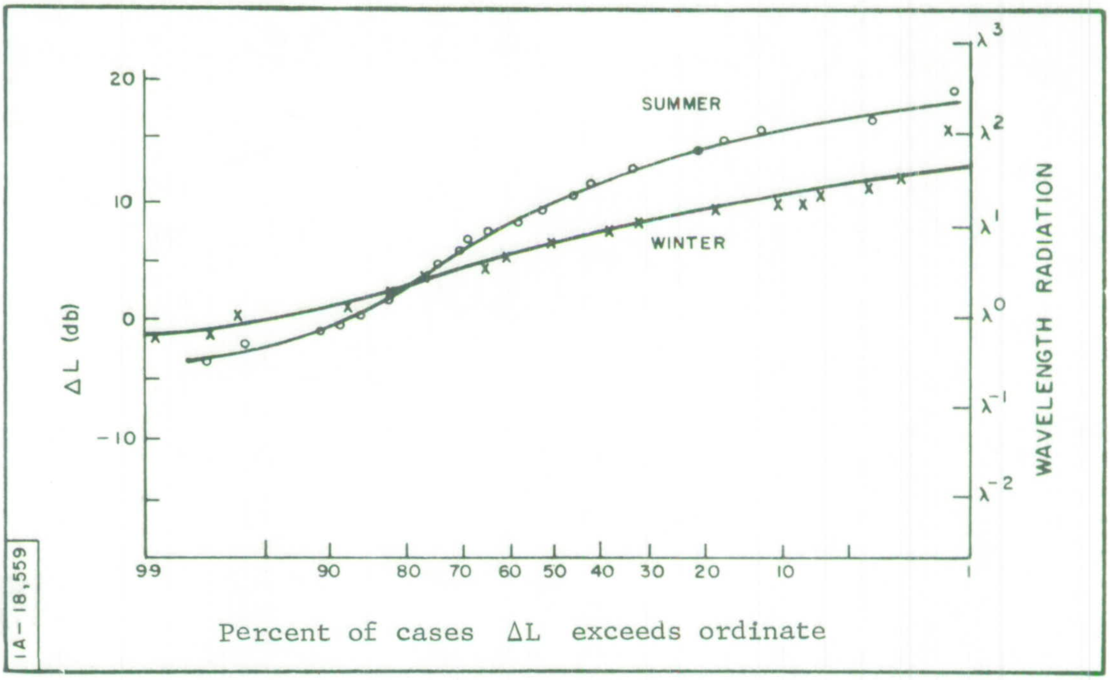

Figure 4. Seasonal Variation in Distribution of Wavelength Dependence

Differences in measured hourly median basic transmission loss, ΔL, scaled frequency experiment, 417.05 Mc/s versus 2290 Mc/s. Round Hill--Crawford's Hill: Winter -- February + March 1957, 76 cases

Summer -- June + July, 80 cases

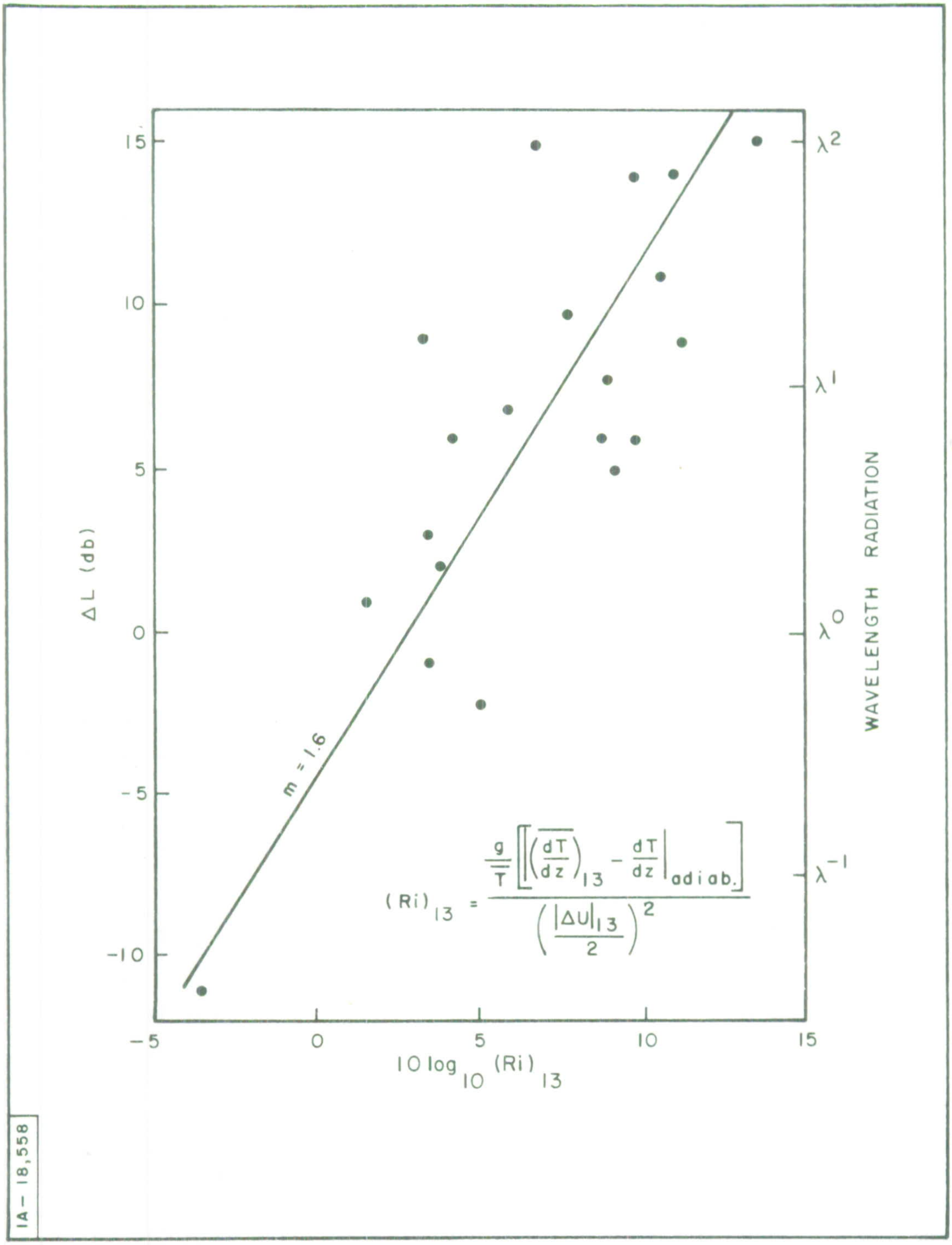

Figure 5. Variation of Wavelength Dependence with Dynamic Stability of the Atmosphere in the Scattering Volume
$r = 0.80$

TABLE III

Some Characteristics of Scatter Propagation

Type / Height of Scattering Volume	Propagation Distances	Distance Dependence	Frequency Dependence	Fading Rate	Polarization	Bandwidth
Tropospheric 0 to 10 km.	100 to 400 statute miles	90% group-power loss 57 db below free space at 100 miles; 12 db/100 miles beyond	Typical values 10 db drop from 50 to 3300; 8 db from 1250 to 9375 mc/s.	Slow component 10 min to 1 hr (8 to 16 db) Fast component 0.1 to 10 c/s (6 to 16db)	Polarization preserved 90%	Booker-Gordon $30/d^3$; Voge $18/d^3$; Staras $4/d^3$; AFCRL $4/d^3$ at 915 mc/s (d in 100 miles)
Ionospheric 65 to 100 km.	600 to 1200 miles	200 - 210 db total loss at 600 miles; flat 500-1100 miles; 10 db/100 miles each side	Scattered power proportional to f^{-8}	Slow component 1-3 hrs; Fast component order of 1 c/s	Elliptical scattered signal; Horizontal polarization customary	About 10 kc/s

(after Nawrocki and Papa)

2. The received signal is due to coherent partial reflection (normal mode) from a refractive index stratified under gravitational interaction.

While it is believed that both of these concepts do pertain at times, in varying degree, the scatter concept will be given more weight here since it dominates current thinking. There is some justification for this circumstance, since the scatter concept has been developed to a far more sophisticated stage and can provide such system paramerters as fading rate and bandwidth. Of course,the values to be assigned to these parameters are ad hoc, but, in principle, the theory does have this extended capability.

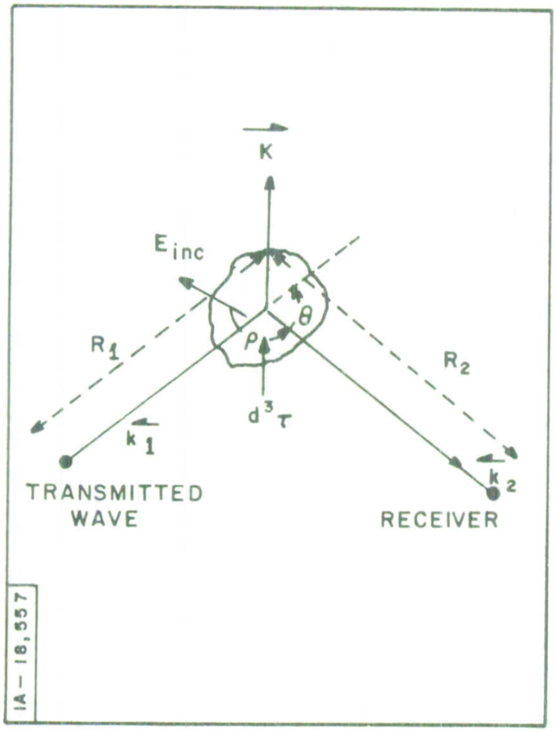

Figure 6. Basic Scatter Geometry

Legend:

R_1 = distance from scattering volume to the transmitter

R_2 = distance from scattering volume to the receiver

E_{inc} and E_s are wave amplitudes of incident and scattered waves

V includes all volume elements which contribute to the scatter

λ = wavelength of radiation

θ = angle through which the scattering proceeds

ρ = angle between the direction of propagation of scattered wave and E_{inc}

Figure 6 shows the scatter geometry that determines the boundary conditons for solutions of the wave equation. For a medium of dielectric constant

$$\epsilon(\vec{r}, t) = \epsilon_o + \Delta\epsilon(\vec{r}, t) , \tag{32}$$

Balser [Ref. 15] has shown that the first-order wave equation is

$$\nabla^2 E_j + k_o^2 \left(1 + \frac{\Delta\epsilon}{\epsilon_o}\right) E_j = 0\,, \tag{33}$$

where $$k_o^2 = \omega^2 \mu_o \epsilon_o\,,$$

$$\frac{\Delta\epsilon}{\epsilon} << 1\,,$$

and E_j indicates a rectilinear coordinate.

The solution of this quasi-static wave equation, subject to the appropriate boundary conditions, gives the following tabulated parameters.

Scattered Field Strength

$$E_s(R_2) = \frac{k_o^2}{4\pi\epsilon_o R_1 R_2} \int_V E_o \sin\rho\,\Delta\epsilon(r, t) \exp(i\vec{k}\cdot\vec{r})\, d^3 r \tag{34}$$

where $\vec{k} = \vec{k}_1 - \vec{k}_2$

Correlation Function

$$C(\vec{R}) = \overline{\Delta\epsilon(\vec{r}_1)\,\Delta\epsilon(\vec{r}_2)} / \overline{(\Delta\epsilon)^2} \tag{35}$$

where a bar over a term denotes the mean value.

$$= \frac{1}{V\,(\Delta\epsilon)^2} \int_V \Delta\epsilon\,(\vec{r})\,\Delta\epsilon\,(\overrightarrow{r+R})\,d^3\,r$$

$$= \frac{1}{2\pi^3\,(\overline{\Delta\epsilon}^2/\epsilon_o^2)} \int S\,(\vec{k})\,e^{i\vec{k}\cdot\vec{R}}\,d^3\,k$$

Spectrum Function

$$S\,(\vec{k}) = \int_V e^{i\vec{k}\cdot\vec{R}}\,C\,(\vec{R})\,\overline{\Delta\epsilon^2}/\epsilon_o^2\,d^3R \tag{36}$$

Spectrum Density Function

$$\Phi\,(\vec{k}) = S\,(\vec{k})\,k^2/\,2\pi^2 \tag{37}$$

Scattering Cross Section

$$\sigma\,(\theta,\,\lambda,\,\rho) = (R_2^2/V) \left| \frac{\overline{E_s\,(R_2)}}{E_{incid}} \right|^2$$

$$= \pi^2/\lambda^2\,\sin^2\rho \int_V e^{i\vec{k}\cdot\vec{R}}\,\frac{\overline{\Delta\epsilon^2}}{\epsilon_o^2}\,C\,(\vec{R})\,d^3\,R \tag{38}$$

Scattered Power

$$P_R/P_T = \frac{\lambda^2}{16\,\pi^2}\,K \int_V \frac{G_T\,G_R}{R_1^2\,R_2^2}\,\sigma\,(\theta_1,\,\lambda)\,d^3\,r \tag{39}$$

where $K = (1 + |P|)^2$

P = the reflection coefficient of the ground at grazing incidence

G_T = normalized power gain of transmitting antenna

G_R = normalized power gain of receiving antenna

While acknowledging the great flexibility and value of the Booker-Gordon theory, its ad hoc nature is of some significance and worth stating here. When the theory was first proposed, the scale of the typical fluctuation as measured at the University of Texas was of the order of 58 meters. This is apparently a derived figure somewhat dependent upon the correlation function assumed. Insertion of the measured $\Delta\epsilon/\epsilon$ for inhomogeneities of this size, together with an anticipated Gaussian correlation function, led to a TSM signal which was orders of magnitude smaller than that observed. Consequently, Booker and Gordon decided to select an exponential correlation function (this yields a higher TSM signal) and to postulate a heirarchy of smaller fluctuations (microturbulence) supported by the larger measured scale values. Opponents hastened to point out the heuristic difficulties with the exponential correlation function (the existence of a cusp) and further noted the convenience of the microturbulence. The latter could account for the measured field intensity, and, at the same time, was of such a nature that its existence could not be checked experimentally. Nevertheless, microturbulence was a step in the right direction.

When it was also found that obviously quiet atmospheres gave significantly higher signals (even the transmission of UHF-video over path lengths of 200 miles), a further ad hoc adjustment was required. This was provided by Bolgiano (see Figure 7) who varied the spectral shape and the point of viscous cutoff η in the refractive index fluctuation spectrum.

This ad hoc character of the theory is in itself no great flaw. However, it is thought that Booker and Gordon have over-estimated the contribution of single scatter in the common volume. Consequently, their concept has some elements quite unreconcilable with the dynamics of the atmosphere. For example, it led quite naturally to Bolgian's suggestion that η varied temporally, lying in the meter region much of the time. This is contradicted by the meteorologists who estimate η to be of the order of millimeters, a result motivated by our own considerations of tying the ever-present microfluctuations to the variations in the Maxwell-Boltzmann distribution. It is not entirely fortuitous that this same size microscale is motivated by the plasma physicists as that (basically the Debye length) appropriate to the ionosphere. As a possible motivation for this it is suggested that, in a neutral corpuscular medium, molecules comprising a microfluctuation interact through matter waves until a dimension of the order of a millimeter is reached. In a plasma such as the ionosphere, interdependence

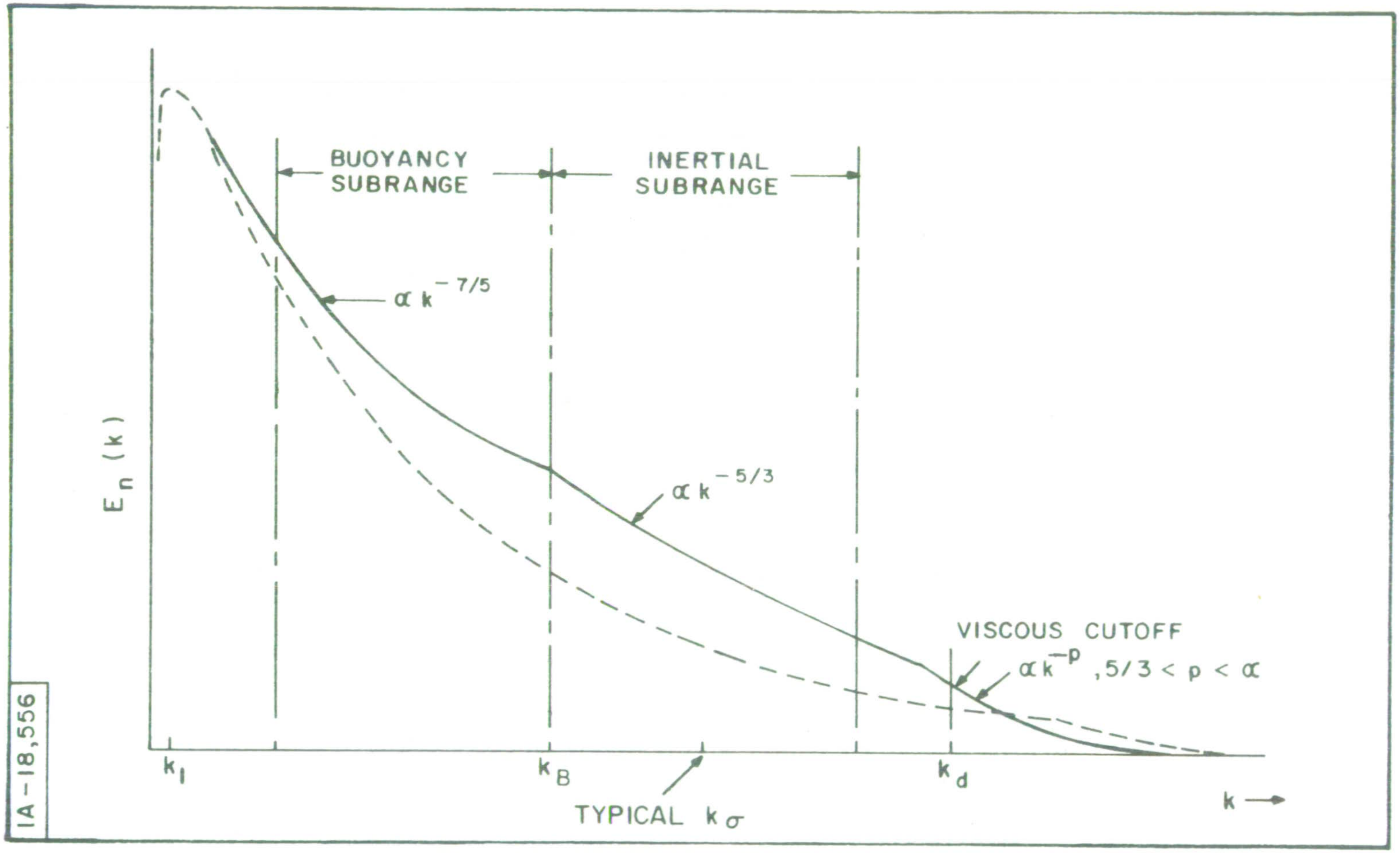

Figure 7. Spectrum of Mean Square Refractive Index Fluctuations

through coulombic interaction (virtual electromagnetic waves if you like) holds until a similar value is attained.

The nonphysical nature of Booker's view on turbulence has led to the suggestion of other correlation functions (see Table IV). However, each of these models preserves the concept of the common volume. The next section shows that the observed everpresent exponential decay is consistent with a concept requiring neither turbulence nor stratification, but requiring the active volume to include the entire region from the optical horizon to the receiver.

Hamilton Statistical Optics

An attempt to devise statistical optics was influenced by the optical-mechanical analogy and an intuitive appreciation of the propagation process. However, analysis of the scatter theory of propagation led the author to take exception to the concept that only the volume common to the unscattered transmitting and receiving beam is important. This may be acceptable for a localized turbulent medium such as a rocket plume or, to a lesser degree, ionspheric scatter, but for the case of tropospheric scatter <u>both the transmitter and receiver are enclosed in the optical instrument.</u>

Intuitively, then, it is the author's belief that:

(1) The received signal is the sum total of rays that extend from T to R.

(2) Each ray follows a path as described by Hamilton's equations.

TABLE IV

Correlation Functions and Spectrum of Turbulence

Description	C (R)	$S(K) / \frac{\overline{\Delta\epsilon}^2}{\epsilon_o{}^2} = \sigma \cdot \frac{\lambda^4}{\pi^2}\left(\frac{1}{\frac{\overline{\Delta\epsilon}^2}{\epsilon_o{}^2}\sin^2\xi}\right)$
Exponential (Booker-Gordon)	$\exp\left(\frac{-\lvert R\rvert}{\ell_o}\right)$	$\frac{8\pi\,\ell_o{}^3}{\left(1 + k^2\,\ell_o{}^2\right)^2}$
Gaussian	$\exp\left(\frac{-R^2}{\ell_o{}^2}\right)$	$\pi^{3/2}\,\ell_o{}^3 \exp\left(\frac{-k^2\,\ell_o{}^2}{4}\right)$
Cauchy	$\left(1 + \frac{R^2}{\ell_o{}^2}\right)^{-2}$	$\approx \frac{8\pi}{\ell_o k^4}\left[1 - \frac{(k\,\ell_s)^4}{(1 + k^2\ell_s{}^2)^2}\right]$ if $k\,\ell_o > 1$
Modified Exponential	$\exp\left(-\frac{\lvert R\rvert}{\ell_o}\right) - \left[\left(\frac{\ell_s}{\ell_o}\right)\exp\left(\frac{\lvert R\rvert}{\ell_s}\right)\right]$	$\frac{6\pi^2\,\ell_o{}^3}{(1 + k^2\ell_o{}^2)^{5/2}}$
Bessel	$\left(\frac{R}{\ell_o}\right) K_1 \left(\frac{R}{\ell_o}\right)$	$\approx \ell_1\,\ell_2\,\ell_3\,\frac{8\pi}{(1 + q^2)^2}$
Anisotropic	$\exp\left\{-\left(\frac{x^2}{\ell_1{}^2} + \frac{y^2}{\ell_2{}^2} + \frac{z^2}{\ell_3{}^2}\right)^{1/2}\right\}$	$S(k) = \left(\frac{\overline{d\epsilon}}{dh}\right)^2 \left(\frac{2\pi}{k}\right)^5$
Mixing-In-gradient	$\left[\overline{(\Delta f)^2}\,R^2\right]$	
Obukov Mixing Theory	$\left[B_f{}^2\,R^{2/3}\right]$	$\frac{S(k)}{\overline{\Delta\epsilon^2}/\epsilon_o{}^2} = \frac{4\pi^2}{3}\,\frac{k_o{}^{2/3}}{k^{1/3}}$ where $k_o = \frac{1}{\ell_o}$

(3) There are <u>no</u> discontinuities in the medium. The refractive index n is a continuous function everywhere. The atomistic nature of a gas provides for finite incremental changes in index, but these are replaced by a fictitious fluid of continuous index which provides the same ray trajectory.

In constructing a statistical theory of rays, the form of the ray Hamiltonian (figuratrix $\Omega(x_k, \sigma_k, t) = 0$) will be considered to be homogeneous of degree 1, so that the temporal parameter u is the phase.

Introducing phase space Φ and density function D, the total number of rays is

$$N = \int D \, d\Phi ,$$

where $$D = D(x_k, \sigma_k, t) .$$

If, further, $D = D(\Omega)$, then, on insertion of the canonical equations

$$\frac{\partial \Omega}{\partial x_k} = - \dot{\sigma}_k$$

$$\frac{\partial \Omega}{\partial \sigma_k} = \dot{x}_k , \qquad (40)$$

there results

$$\frac{dD}{dt} = \frac{\partial D}{\partial \Omega} \frac{\partial \Omega}{\Omega x_k} \dot{x}_k + \frac{\partial D}{\partial \Omega} \frac{\partial \Omega}{\partial \sigma_k} \dot{\sigma}_k + \frac{\partial D}{\partial t}$$

$$= \frac{\partial D}{\partial \Omega} \frac{\partial \Omega}{\partial x_k} \frac{\partial \Omega}{\partial \sigma_k} - \frac{\partial D}{\partial \Omega} \frac{\partial \Omega}{\partial \sigma_k} \frac{\partial \Omega}{\partial x_k} \equiv 0 \ . \tag{41}$$

Consequently, if the density function in phase space is a function only of the ray Hamiltonian, then D is stationary, and Liouville's theorem pertains.

This time (or rather phase) stationary ensemble of rays is assumed in the motivation of a propagation law. The time-stationary mechanical ensemble is the Boltzmann distribution

$$P(H) = C e^{-H/KT} = C e^{-E/KT} , \tag{42}$$

where $E = p^2/2m$ are the eigenfunctions (here continuous) of the Hamiltonian H. In pursuing the analogy, there is no difficulty in replacing derivatives of H by the corresponding derivatives of Ω, but the direct substitution of Ω for H produces difficulty since it has been assumed that the relation $\Omega(x_k, \sigma_k, t) = 0$ always holds. However, the concept of a probability function $P(\Omega) = P(x_k, \sigma_k)$ intuitively exists, and it is, uniquely determined by the motion of particles comprising the Boltzmann ensemble. Since only the time-stationary case is being considered, no turbulence or spectrum of

eddies are present. Only the time-stationary particle distribution is involved.

Assuming a very large number of molecules N enclosed in a fixed volume V, let v be a fixed subvolume large enough to hold a large number $[m(o \ll m \ll N)]$ of molecules. The mean value of m in v is $m_o = pN$ and the actual value is $m = m_o + y$. Then, by the Bernoulli theorem,

$$f_N(y) = [2\pi m_o(1 - m_o/N)]^{-1/2} \exp\left\{- \frac{y^2}{2m_o(1 - m_o/N)}\right\}. \tag{43}$$

If $N/m_o = \infty$, the customary relation

$$f_\infty(y) = (2\pi m_o)^{-1/2} \exp\left(- \frac{y^2}{2m_o}\right) \tag{44}$$

results; but, since N is finite, there are two contributions with standard deviations

$$\left(m_o - \frac{m_o^2}{N}\right)^{1/2} ; \quad m_o^{1/2}.$$

From these particle distributions, the phase refractive index figuratrix will exhibit mean values a_i (termed eigenvalues of the figuratrix)

$$\frac{\sigma_x^2}{a_x^2} + \frac{\sigma_y^2}{a_y^2} + \frac{\sigma_z^2}{a_z^2} - 1 = 0 \ , \tag{45}$$

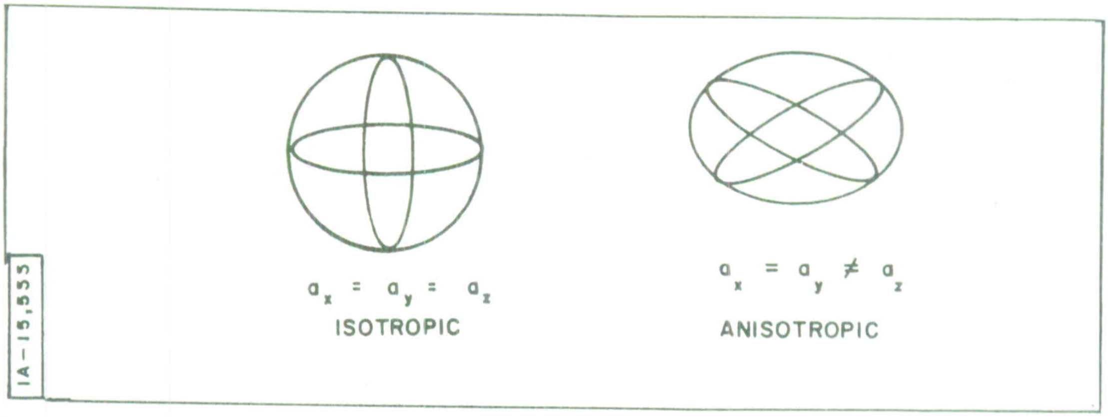

Figure 8. Figuratrices

and deviations Δa_i which depend upon

1. The total number in the local region, $v_{o,}$ (Gaussian fluctuation).
2. The number of weighted constituents such as H_2O molecules (Gaussian fluctuation) in the local region, v_o .
3. The velocities of the particles (Boltzmann distribution of energies $e^{-mv^2/2KT}$, again Gaussian distribution of velocities).

The sum total of these is a Gaussian distribution in refractive index eigenvalues a_i .

Gravitation can be included to give density dependence on altitude, and temperature can be arbitrarily imposed according to

the customary lapse rates. There is one further imposition on the fluctuations in phase: the phase fluctuation cannot assume such a value that the related group velocity would exceed the velocity of light; i.e.,

$$v_{group} = \frac{c}{n_{group}} = \frac{c}{n + \omega \dfrac{\partial n}{\partial \omega}} \leq c \; . \tag{46}$$

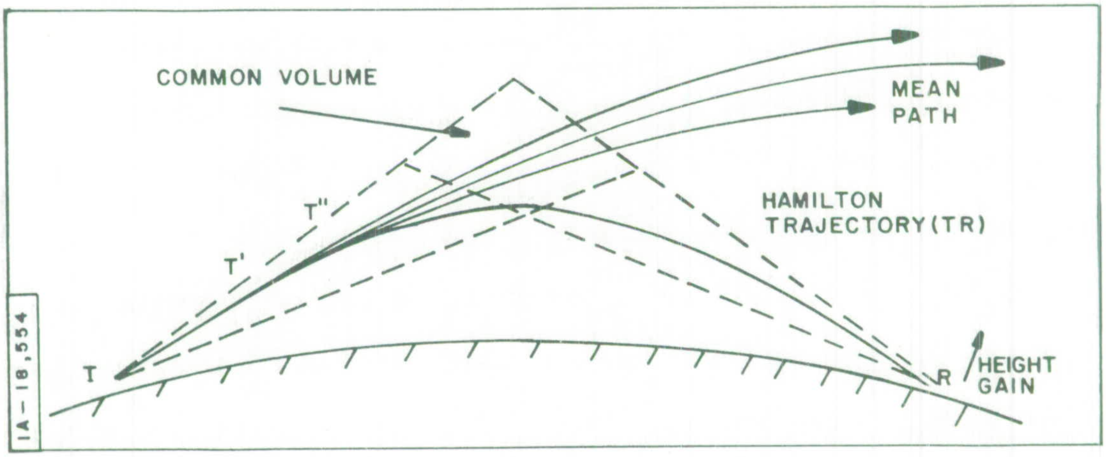

Figure 9. Illustration of Beyond-The-Horizon Transmission

Figure 9 shows the mean path for an air mass under gravitation, and tries to illustrate how the Hamilton rays starting at T actually fan out, each ray becoming exponentially less probable as the deviation from the mean index increases.

If it is assumed that the rays arriving at R follow a path of constant curvature (constant deviation Δa_i), then the probability of arriving at R is proportional to

$$\prod_{i=1}^{N} A_i \, e^{-K(a_i)\Delta s_i} \approx A_i^N \, e^{-ks} = 1 e^{-ks} . \qquad (47)$$

This is exactly the expression for the average everpresent signal. Idempotency of the factors A_i^N apparently implies that the overall fluctuations are not being conserved. Actually, the concept of a constant curvature is an additional postulate because it is clear that Hamilton's rays exist where the curvature does change. Nevertheless, the theoretical view predicts that, even in a constant-temperature time-stationary medium, there exists a great many small-scale fluctuations. Although there is no obvious physical mechanism in the neutral gas that can play the role of the Debye length in ionized media, the mean fluctuation is measured in millimeters and the refractive index deviation in the Gaussian distribution is basically proportional to the density, (Figure 10). Consequently, the idea of an

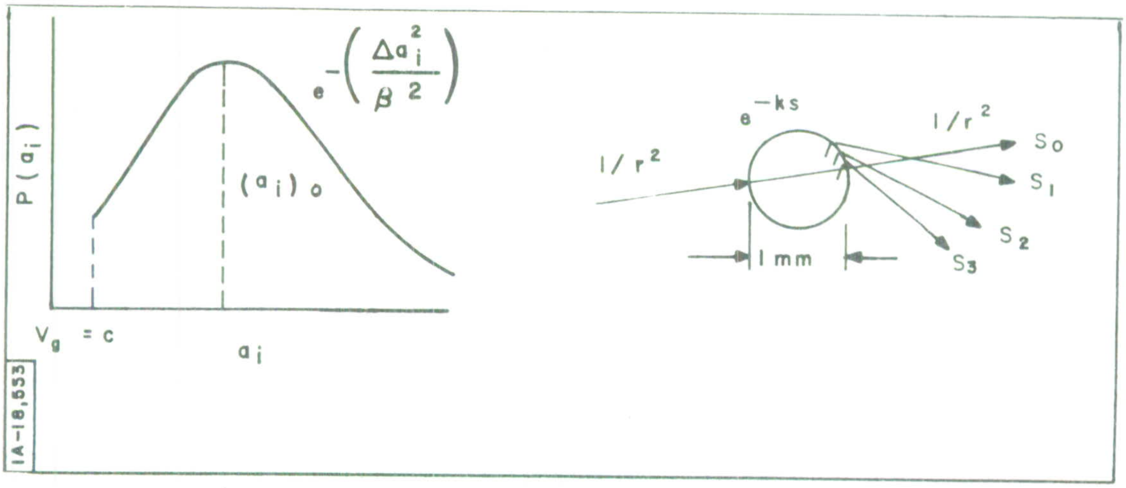

Figure 10. Two Representations of Exponential Loss in Scattering

average curvature for the off-the-median trajectory is logical enough, since no gravitational separation in the troposphere is assumed and the lower part of the troposphere is not properly weighted.

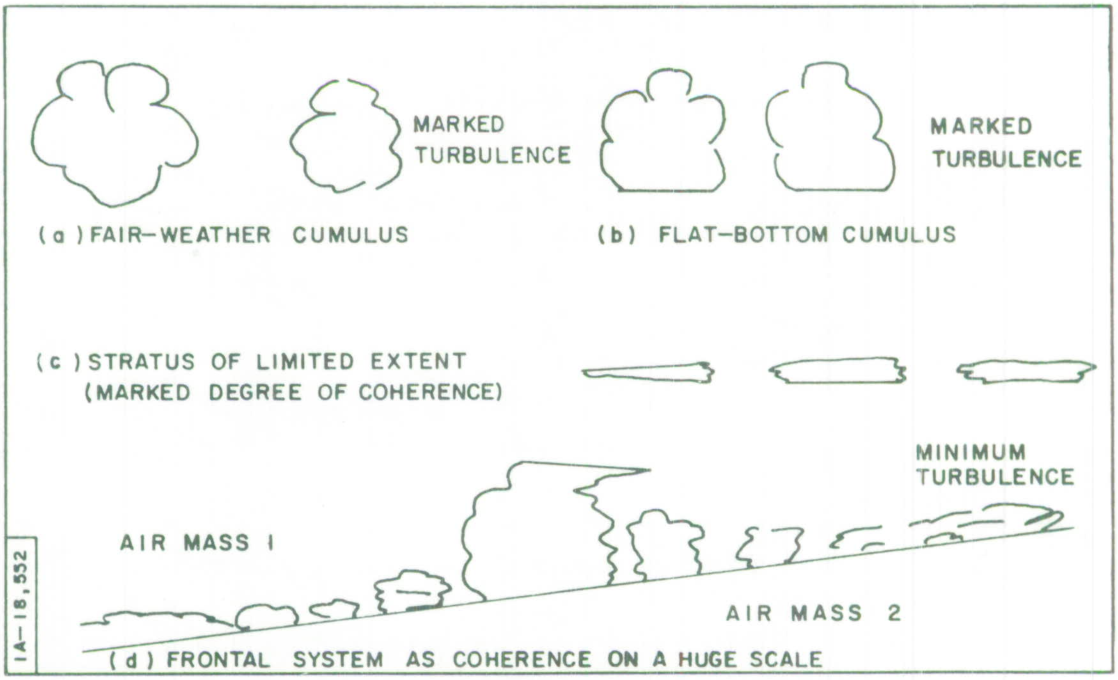

Figure 11. Meteorological Conditions Exhibiting
A Varying Degree of Coherence

In Figures 11 (a through d), cloud forms exhibit an increasing degree of coherence. Both cohered refraction and partial reflection, Carroll-Ring [Ref. 16], and turbulence, Booker-Gordon, can then materially alter the signal characteristics. In spite of the apparent similarity between the Hamilton motivated results and the scatter theory employing a Gaussian correlation function, there is a basic difference due to the assumed size of the scattering volume.

When an isotropic single scatter in the medium common to the two beams is assumed, Φ is an ignorable coordinate in the scattered amplitude. Although the author suggested a decade ago that such a Φ-dependence should be expected, the interpretation of a Φ-dependence, if found, is undoubtedly ambiguous.

There are of course more meaningful tests of the validity of the intersecting-volume model.

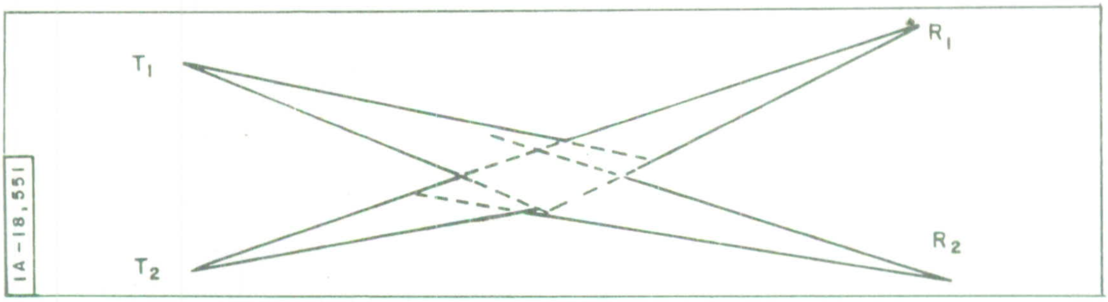

Figure 12. Cross-Scatter Beams With Common Volume

1. Crossed beams (Figure 12) are adjusted to intersect a common volume V. Existing scatter theories would indicate a high correlation between simultaneous (in the Newtonian sense) signal intensities and fading rates. The Hamilton ray theory would predict the low degree of correlation, approximately equal to the fraction, $\dfrac{\text{common path}}{\text{total path } (\sim HR)}$.

2. The Hamilton theory states that the rays retain phase information; therefore, this phase information can be regained. Ideally, for the common-volume theories, the experiment is given schematically in Figure 13.

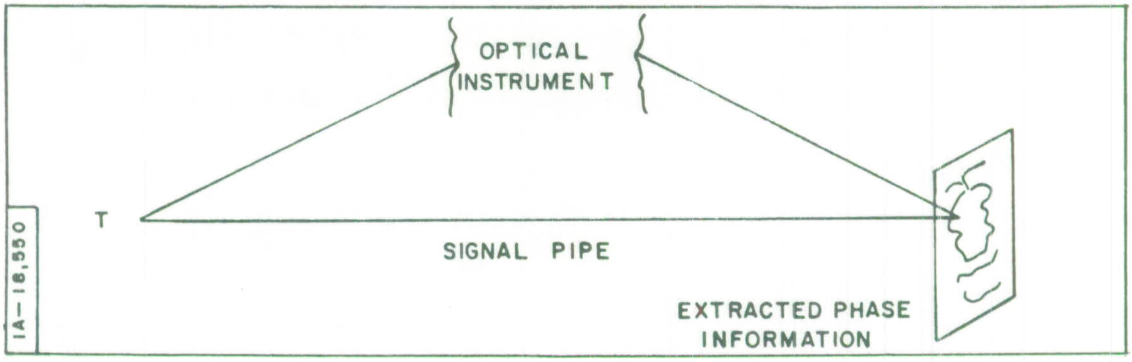

Figure 13. Technique for Evidencing Phase of the Optical Instrument

If an optical frequency (laser) were used, the resultant of the scattered and unscattered signals (arranged to be of the same mean amplitude) could be photographed and detailed information could be derived concerning the optical instrument. For microwaves, conceivably the screen could be replaced by a large matrix of silicon diodes, but unfortunately, the signal levels (70 to 100 db below free-space levels) are too low to be readily detectable by such a means.

The Hamilton-ray approach, even without extending its sophistication, has been of value in evaluating the various scatter theories. The investigation has motivated several changes in the Booker-Gordon theory. These changes are described below.

First, turbulence is not necessary to provide a scatter signal. Even a constant-temperature homogeneous gas has the everpresent microstructure (millimeter fluctuations) capable of providing beyond-the-horizon transmission.

Second, all peripheral considerations support the choice of a small-scale Gaussian correlation over the expotential selected by Broker and Gordon. For contributions of turbulence to the scatter signal, the Obukhov-Silverman [Ref. 17] construct is preferred to that of Booker-Gordon.

Third, the model of the intersecting volume, while possessing great simplicity, cannot provide for the observed exponential propagation law, e^{-ks}. Only a large number of successive small-angle refractions or scatterings in the entire volume can do this, especially those between the optical horizon and the receiver.. This concept is supported by consideration of the inverse scattering problem because the reflected signal provides for a unique dielectric.

Hamilton Backscatter

So far, the propagation medium has been treated as being contintinuous, and, this has produced a picture of the forward-scattering process. If the process is extrapolated to backscatter, the interpretation of the ray making a 180° turn does not appear to be intuitively a very satisfying concept. Consequently, it would be desirable to readmit the existence of discontinuities where the Hamilton jump conditions $\Delta\sigma_k \, \delta x_k = 0$ provide for the mean ray. The backscattered signal can then be obtained by the construct developed by the New York University group. The radar return is the sum of extrema TT obtained by wrapping Hamilton rays about each discontinuity.

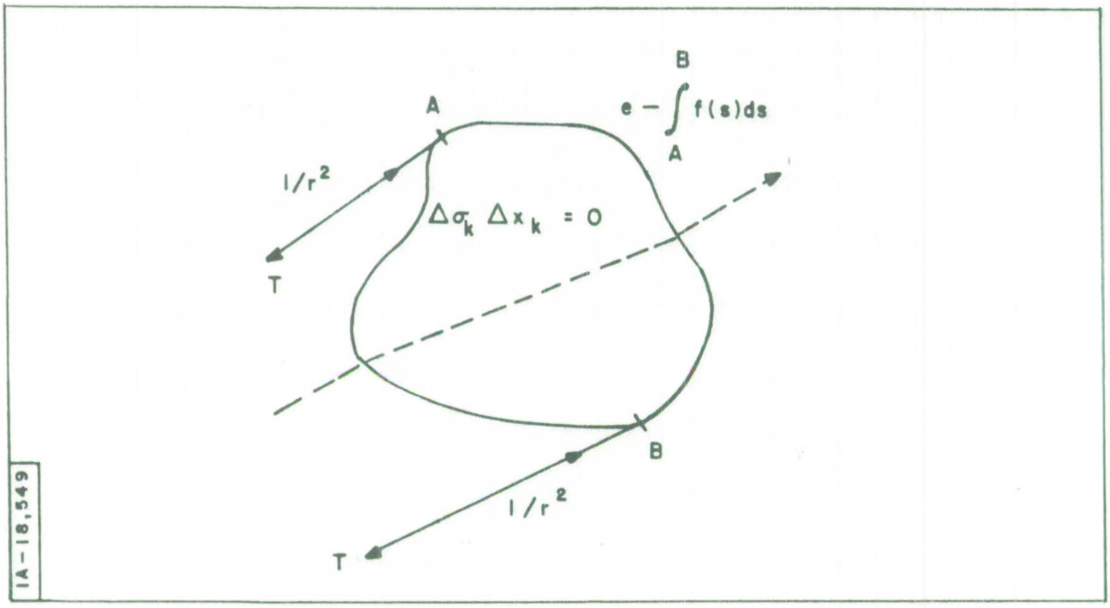

Figure 14. Ray Configuration for Backscatter

In figure 14, an inverse-square attenuation is taken from T to the tangential points A and B. From A to B (and B to A) an exponential loss is imposed. This mode will not be discussed in detail, since plasma physics (interactions of waves and particles) has already given a satisfactory solution for the most interesting application, the ionosphere. However, there has been an unwarranted inclination among radio propagationists to give credence to Gordon's non-physical concept of individual electron backscattering macroscopic waves $(\lambda > \lambda_D^* > r_e)$ with a cross section consistent with a unit albedo spherical reflector of classical radius r_e. Such a concept is fraught with difficulties for the interdisciplinary physicist.

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* λ_D is the Debye length.

Without elaboration, the author merely reasserts a position taken in 1958 that, in this radar case, everpresent Gaussian fluctuations in electron density provide the scattering mechanism. As the Hamilton trajectory theory indicates, a further decrease in the wavelength until $\lambda > \lambda_D$ does not provide for a radical change in the propagation mechanism. It is somewhat paradoxical that Gordon abandons the Booker-Gordon model in that example where it is least inappropriate. Indeed, the observed radar cross section can provide a test of the appropriateness of their model of intersecting beams in the forward-scatter case.

SECTION IV

QUANTUM OPTICS

Quantum theory is essentially a discipline based on the DeBroglie concept that all microscopic extracules exhibit both corpuscular and wave characteristics. Consequently, the quantum theory of both particles and waves has a common element. Indicative of this, Dirac [Ref. 18] has shown that the Schrodinger representation for a set of equivalent particles is identical to the quantum field representation of a system of waves. In at least one direction these ideas have been elegantly developed. In the inverse-scattering problems (determining the character of a dielectric from the reflection coefficient), Kay and Moses [Ref. 14] employ mapping of the electromagnetic wave equation,

$$\nabla^2 E_z(x) + \mu_o \epsilon(z) \frac{\omega^2}{c^2} E_z(x) = 0 , \tag{48}$$

into the unidimensional wave equation

$$\left[- \frac{d^2}{dx^2} + V(x) \right] \Psi(x) = p^2 \Psi(x) . \tag{49}$$

The Gelfand-Levitan algorithm is used to compute the potential function and the wave function from the scattering coefficient, and mapping provides the solution of the electromagnetic problem.

In theory, the backscatter problem discussed in the previous section could be treated by starting with the <u>observed signal</u> and

developing the character of the dielectric, but the necessary condition of a continuous dielectric constant may be overly restrictive. Fortunately, the approach that motivates a dielectric from the point of view of the plasma physicist and then derives the backscattered signal has provided an excellent solution.

Because of these physical considerations, the synthesis of quantum optics by analogy could not be expected to lead to anything new. Nevertheless, on cursory inspection of the canonical nature of x_k and σ_k it might be thought that there is a limit to the ability of an observer to know simultaneously both the position and slowness of a ray. From the quantum mechanical relations

$$p_k = - i\hbar \frac{\partial}{\partial q_k}$$

$$p_\ell q_k - q_\ell p_k = - i\hbar \, \delta_{\ell k}$$

$$\dot{p}_k = \frac{i}{\hbar} [H, p_k], \qquad (50)$$

the following could immediately be written

$$\sigma_k = - i \frac{\hbar}{E} \frac{\partial}{\partial x_k}$$

$$\sigma_\ell x_k - x_\ell \sigma_k = - i \frac{\hbar}{E} \delta_{\ell k}$$

$$\dot{\sigma}_k = \frac{iE}{\hbar} [\Omega, \sigma_k] \tag{51}$$

Considerations of dimensional identity demand inclusion of a factor with the character of the inverse of energy, while the physics of the problem imply uniquely that this energy must be that of the ray, $E = \hbar\omega(N)$. Consequently, Planck's constant vanished from the commutation relation

$$\sigma_\ell x_k - x_\ell \sigma_k = - \frac{i}{n\omega} \delta_{\ell k} .$$

Usually, the argument is reversed so that Hamilton's relations in optics and mechanics lead to quantum theory, provided that the Planck relation $E = \hbar\omega$ is assumed.

For purposes of this discussion, the basic characteristics are revealed in a comparison of the uncertainty principle for mechanics

$$\Delta p_k \Delta_\ell = h \delta_{\ell k} , \tag{52}$$

and the analogous expression for optics

$$\Delta x_k \Delta \sigma_\ell = \frac{\hbar}{E} \delta_{k\ell} = \frac{\hbar}{Nh\omega} \delta_{k\ell} = \left(\frac{\delta_{k\ell}}{Nc}\right) \lambda . \tag{53}$$

This demonstrates that, in a sense, quantum theory was implicit in the classical notion that the energy of a ray of known frequency was

spread over the wavelength λ. As may have been anticipated, quantum optics of the ray type is then an essential redundancy.

This section concludes with several pertinent peripheral considerations. First, in the concept of quantum optics, the author has attempted to push the discipline past the limitation that λ must be large compared to atomic dimensions. Actually, as λ decreases, atomic collisional processes such as photoionization become important. Second, only geometric or ray optics have been considered. The material does not bear at all on the subject of quantum field theory where operators can be introduced for the electromagnetic field variables, E, H, B, D via canonical potential functions Ψ, Π. Third, the theory is linear; ray intensities are so low that no change in the refractive index is introduced. This automatically eliminates such phenomena as high-intensity rf or laser beams where ionization (breakdown) may be produced even though $\hbar\omega$ is not sufficiently large to accomplish ionization directly (intermediate activated states play a prominent role).

Finally, the conclusion that, in essence, quantum mechanics combines the concepts of classical mechanics and optics is neither entirely trivial nor barren. For small systems where the limit to observation is relevant, Dirac's [Ref. 18] concept of "superimposability of translational states" was considered. To explain the experimental results from interference in double slits, interference in split beams, and polarized photons at oblique incidence in

tourmaline crystals, Dirac suggested that translational states (for example, photons in transit between observable interactions) carry what are classically considered as mutually exclusive parameters. In the split-beam experiment, for example, roughly half of the photon traverses each beam and the photon then interferes with itself on the subsequent superposition of the two beams.

It is true that if physics is defined as that discipline appropriate to the observable world (as indeed the author does) then Dirac's invention is unimpeachable; for if a device is introduced to identify which beam the photon is really (observably) in, then the interference effect automatically and quite logically vanishes. However, Dirac's ingenious construct is an invention hardly to be labeled as a subjective truth. Personally, the author finds it disconcerting to admit that, even beyond the ken of observation, a photon <u>must be one</u> (a corpuscle) <u>yet cannot be one</u> (follows two different paths). The gymnastics required for pursuit of quantum optics now bear some fruit since it suggests a model different from that of Dirac's superimposable translational states. If quantum mechanics is indeed the combined optical-mechanical discipline, then all wavicles (e.g., photons and electrons) are associated with Schrödinger waves. For the electron, Heisenberg [Ref. 25] gives as the velocities of the phase and group waves the values

$$v_\phi = \frac{E}{p} = \frac{p^2/2m}{p} = \frac{p}{2m} \; (= v/2) \, ,$$

$$v_g = \frac{dE}{dp} = \frac{p}{m} \; (= v) \, ,$$

where E is the kinetic energy of the electron.

Thus, the phase wave associated with the wavicle has the velocity equal to one-half the group wave value (v_g is of course the velocity of the corpuscle) in contradiction to the author's understanding of quantum theory. Instead of v_ϕ being a function of the kinetic energy, the author understands function of the total energy, i. e., $v_\phi = \frac{c\sqrt{m^2c^2 + p^2}}{p} \geq c$. Since, the phase wave of a wavicle possessing rest mass is always greater than the characteristic velocity, the phase wave can be used as the carrier for the probability indicator required for translational states. Each time the wavicle interacts with matter, the information is impressed on the phase wave. The wavicle only goes <u>one</u> path (with the group velocity v_g) but in the subsequent or second interaction, the new-distorted phase wave interacts with its new boundary condition to produce the phenomenon of self-interference. If a third boundary condition is placed somewhere in the path between incidents 1 and 2, then the phase wave is further distorted, so that the simple phase relation of incidents 1 and 2 necessary for 1, 2 self-interference no longer exists. At all times, a photon or electron (ignoring coulombic forces) interacts with itself via its own phase wave. The result is the same as that achieved by Dirac's construct, but the basic self-contradiction in the world-beyond-observation is removed.

Peripherally, the concept of a phase velocity in excess of the characteristic velocity obviously does not violate relativity, since the observable signal is carried at the group velocity (still the velocity of the wavicle).

These considerations support the notion that the quantum-mechanical t can be either phase or time, the proper choice being dependent upon the specific problem. No light has been cast upon the question of the appropriateness of spatial isotropy for nucleons inside the nucleus.

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13. ABSTRACT

Synge's formulation for Hamilton's theory of geometric optics is used to specify the rays for a variety of problems. Based upon the analogy between ray and particle trajectories, an attempt is made to treat the problem of beyond-the-horizon atmospheric propagation. It is shown that the observed propagation law (e^{-ks}) can be motivated from the concept that the received signal is the sum of the Hamilton rays linking the transmitter and receiver. This implies three differences with respect to the Booker-Gordon theory for the everpresent scatter signal: a change in the correlation function from exponential to Gaussian; adoption of millimeters as the size of the characteristic fluctuations; and an extension of the critical volume to include manyfold scatter in the tropospheric path from the optical horizon to the receiver.

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